Understanding mathematical thinking through narrative accounts of syncretic thought

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Starting with the uncontroversial premise that teachers' abilities to foster mathematical thinking relies on their understanding of what mathematical thinking is, this paper sets out, and models, a methodology that proceeds on the basis that such an understanding can be 'risky' and that it can be approached more safely by first understanding syncretic thinking, a kind of thinking that is distinctly un-mathematical. This methodology represents a mode of enquiry designed to contribute to our understanding of mathematical thinking by presenting narrative accounts of syncretic thought's operation in recognisable instances of 'error' in natural human interaction. Examples are included as an appendix.

The importance of mathematical thinking

Worldwide, various regulators and teachers' guilds have set standards that make explicit the expectation that mathematical thinking will be an outcome of a mathematics education (for example, AAMT 2006; DfE 2012; NCTM 2012). These standards define the teacher's role with respect to recognising, modelling and fostering this kind of behaviour. For example, the NCTM's (op. cit.) process standards encourage a practice that promotes the development of pupils' skills with respect to communication, analysis, evaluation, conjecture, reasoning and proof, paying particular attention to conjecture, reasoning and proof, these being considered to be peculiarly mathematical kinds of activity. While acknowledging the need for, and importance of, an understanding the nature and structure of mathematical thinking, I suggest that a direct approach to such an understanding entails certain 'risks'.

- Prior to the development of any higher order or mathematical ways of thinking children are already communicating, analysing, evaluating, conjecturing, reasoning and proving, however idiosyncratic and naïve those thought processes might be. The question, then, is not one of bringing these behaviours into being; the question is how to bring socially sanctioned varieties of these behaviours into being.
- It is not unreasonable that the general public should expect a mathematics educator to know what mathematical thinking is. However, because this

public expectation is so reasonable, and so elementary, the question of the nature and structure of mathematical thinking risks being too sensitive for teachers and researchers to approach with the scepticism, rigour and vigour that they might do with other aspects of their practice. As a consequence, the notion of 'mathematical thinking' risks being treated as a catch phrase or slogan endowed with the mystical potency of an incantation, so that if mathematical thinking is not well understood, then it will tend to remain so as there is too great a personal risk attached to one's not being able to use the concept appropriately in public.

- Any negative effects that result from a lack of understanding of mathematical thinking risk being aggravated by our natural tendency to see paradigms (choices) as being exclusive so that, for example, if mathematical thinking is a good thing then un-mathematical thinking must be bad (which, per se, it is not).

I suggest that these risks can be mitigated if we approach an understanding of mathematical thinking indirectly, via an understanding of syncretic thinking, which is treated here as being a kind of thinking that is distinctly un-mathematical.

The importance of syncretic thinking

Both Vygotskii and Piaget identified syncretic ways of thinking as immature phases in the psyche's development (Piaget 1953; Piaget 1959; Vygotskii 1987). It is taken here that *syncretic thinking*, *syncretistic schemas*, *pseudo-concepts*, and *syncretic complexes* (terms variously used) all refer to the same thing: the child's imitation of others and their immature construal of experience according to its facile features and naïvely inferred causal relations.

Syncretic thinking developmentally precedes and conditions the possibility of higher order thinking and is of an entirely different quality (Vygotskii 1981). While the literature opposes syncretic thinking to higher order thinking, it is taken here that higher order thinking enables mathematical thinking, and that, hence, syncretic thinking is un-mathematical. Arguments in favour of understanding syncretic thinking include the following.

- An important feature of syncretic thinking (pointed out by Vygotskii) is that its functioning is not replaced by the higher order mental functions that enable mathematical thinking. While higher order thinking comes to dominate it, syncretic thought may resurface under suitable conditions.
- The notion of syncretic thinking has an established history that features in the literature on developmental psychology (for example, Piaget 1953; Piaget 1959; Vygotskii 1987). This provides us with a rich and well-recognised source of explanatory material regarding its nature and its relationship with higher order thinking.
- Because syncretic thinking is an immature form of thought that precedes mathematical thinking, mathematics educators should feel that they are at a sufficiently safe psychological distance from this kind of thinking to be able to approach it relatively objectively.
- Recognising that syncretic thinking has a natural place in the human organism's development will reduce the risk of it being treated as being

"wrong" or "bad". The value of understanding syncretic thinking through moments of error complements our efforts to understand pupils' misconceptions (Küchemann 1981; Morgan 1994; Askew and William 1995; Swan 2001).
- Describing syncretic thinking gives us an additional, and less risky, route towards an understanding of mathematical thinking by beginning to systematically rule out what it is not.

The importance of error

Because 'thought' (regardless of how we choose to define it) is hidden from our direct view, any attempt to positively describe 'mathematical thinking' is unavoidably faced with a number of difficulties:

- Although thought may manifest itself in material activity, its functioning is of a different phenomenological kind. This can lead to confused accounts of its operation based on incommensurable objects of enquiry (such as thought, cognition, feelings, intention, purpose, meaning, language, semiosis, communication, and so on);
- Thought 'exceeds' its multi-modal means of manifestation (such as speech, gesture, written prose and poetry, music, song, dance, the visual and plastic arts, and so on), but it can only ever be described by employing these same means, that is, thought can never be described in its own terms;
- Positive attempts to approach mathematical thinking via other psychic functions that are taken to constitute its operation, like 'higher order mental functions', 'deep understanding', 'reasoning', 'recall', 'memory', 'abstraction', and so on, don't resolve our problem, they simply relocate it, and;
- 'Mathematical thinking' is, I suggest, a cultural object and, as such, its definition and critique belongs to aesthetics and all the usual means by which a community comes to decide what something *is* and what is *worth*.

While the difficulties associated with observing mathematical thought apply equally to syncretic thought, it is possible that observed instances of error provide the empirical material with which to reliably observe the operation of syncretic thought, and hence, to objectively approach an understanding of mathematical thinking indirectly, by understanding what it is not.

It hardly needs to be pointed out that 'error' is an extremely valuable pedagogical tool. The pedagogical poverty of 'right answers' is that they reveal nothing of mathematical thought. The outward expression of the processes and products of mathematical thought can be imitated, and un-mathematical thought does not necessarily produce 'wrong answers'. When the product of reasoning is recognisably 'wrong' on the other hand, there is the possibility of 'seeing through' that error into the operation of un-mathematical thought. For example, taking 'proof' as a characteristic of mathematical thought, it is possible to imitate any one of the many proofs of Pythagoras' theorem. It is unlikely however that, in the context of the mathematics classroom, one would knowingly imitate an erroneous proof, unless it was for comic effect (see '7x13=28' in the appendix, for example). That is, I take it that errors are typically authentic, while the behaviour that leads to 'right answers' may or may not be.

The justification, therefore, for a negative approach to mathematical thinking, via syncretic thinking, is the methodological power of the possibility of syncretic thought being manifestly and authentically exposed in recognisable moments of error and/or confused behaviour.

A provisional description of syncretic thought

With respect to the methodology promoted here and elsewhere (Kusznirczuk 2012a; 2012b), syncretic thought can be understood according to its abstract features as described by any given theoretical account. The choice of theory is not important, so long as it is pertinent and passes the usual logical and aesthetic tests. The value of a theoretically grounded description to the methodology suggested here lies in the possibility of the systematic and disciplined organisation of a shared vocabulary, even more than any explanatory power it may possess. The provisional description of syncretic thought's characteristic features, along with natural examples of its operation, forms the basis for a dialectical enquiry, the value of which rests on personal experience and participation in the enquiry rather than the propagation of its results or the pursuit of some teleological end.

Vygotskii describes the characteristic features of syncretic thought genetically, through three phases (Blunden 1997). To avoid the question of the possibility of 'pure' syncretism, I choose to treat the first two of Vygotskii's phases as characterising the operation of syncretic thought. The following, then, are the characteristics of lower orders of thought that precede Vygotskii's notion of 'conceptual thought', his third phase of development, including the factual, scientific, and mathematical kinds of thought that characterise 'higher mental functions'.

- *'Pure' syncretism*: Objects and their relationships are established idiosyncratically according to the child's subjective apprehension of their surface features. The characteristics of this phase of syncretic thought are *trial-and-error* ('pure' syncretism), *egoistic selection* (the inability to apprehend any point of view other than one's own), and, *idiosyncratic combination* (the grouping of objects, according to naïve and egocentrically inferred causal relations, in 'heaps' based on trial and error and/or egoistic selection).
- *Thinking in complexes*: Objects are construed according to a 'mixture' of purely syncretic and mimetic means. Objects and their relations are not only construed superficially and subjectively, they are also construed through the imitation of the behaviour of others and the value placed on 'things' by others, as perceived by the subject, so that this immature construal of experience begins to phenotypically resemble adult conceptions of 'things'. Vygotskii referred to this as the stage of the *pseudo-concept*.

Presenting examples of syncretic thought in narrative form

In keeping with good pedagogical practice, the best way to communicate a complex concept such as syncretic thinking is to present recognisable examples of its operation. This methodology's power, if it has any, springs from the cultural recognisability of the situations its narrative examples present. So, based on theoretical accounts of syncretic thought such as the Vygotskian account given above, the appendix to this paper offers a collection of incidents and examples of behaviour that indicate the possibility of syncretic thought's operation, presented in narrative form.

It must be noted that, in this mode of enquiry, the *presentation* of examples of syncretic thought's possible operation does not analytically follow its *description* in some fixed or isomorphic methodological sequence, the two activities dialectically inform one another, with descriptions 'recognising' presentations and presentations 'realising' examples of their descriptions.

Finally, while the appended examples feature children's syncretic thought, syncretic thought is not restricted to children. To show that immature thought persists into adulthood, some examples of adult behaviour indicating syncretic thought's operation are also included.

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Appendix: Natural examples of syncretic thought

Peek-a-boo
Very small children (less than a year old) love the game of peek-a-boo, where you alternately hide and reveal an object before them. It is possible that this game's surprising effect is due to an aspect of syncretic thought, namely, the child's inability to recognise the persistent existence of objects beyond its immediate field of perception.

Height/age = c
Many school-age children see height and age as being proportionally connected so that the older you are the taller you are, and the taller you are, the older you must be. Two children of the same age and of different heights, if they do not know each other's age, will tend to treat one another as though the taller one were older and the shorter one younger, and vice-versa.

Weight = k/height
For some middle-school children height is (counterintuitively) taken to be *inversely* proportional to weight. Some year 8 girls once explained this relationship to me. My understanding of their line of reasoning is as follows: Models in magazines are tall and skinny (tall = skinny). 'Ordinary' people are short and not so skinny. Now, skinny = lightweight, and so, by a transitive kind of reasoning, tall = lightweight, and, as a corollary, short = heavy, so, the taller you are, the less you weigh.

Speed = proficiency
According to this syncretic conception of mastery, the better you are at doing something the faster you can do it. This reasoning works just as well in reverse so that if you can do something fast then you must be, by definition, good at it. Speed is identified with proficiency. For example, as a young boy, learning to play the piano, proof that I had been conscientiously practicing my assigned pieces was demonstrated (or so I thought) by playing them all as fast as I could.

Bil-car!
My wife's godson has an English father and a Swedish mother. His parents speak to him bilingually. When he first learned to speak, his parents would, for example, point to a car and say "bil" (which is Swedish for 'car'), followed by a short pause, and then "car." So, naturally, whenever he saw an automobile, he would cry out "bil-car!", which is fair enough. The thing is though; he also called the vacuum cleaner "bil-car," and he would sit on it and ride around the house as his mother vacuumed.

How old are you?
Ask a very young English-speaking child how old they are and they will confidently tell you that they are (say) "three". If you ask them "Three what?" then (typically) a blank expression will be your only reply. You can prompt them for a unit of measure with clues such as "are you three balloons?" or "… three ponies?" but they will typically not understand what you want of them. [Note that this test doesn't work in those languages where the unit of measure is included in the formulaic response ("*j'ai trois ans*", for example).]

How far is it to Baba's house?
One summer I was taking my sister's three young children by train to visit their grandmother. As soon as we'd boarded the train and taken our seats my nephew, Cameron, asked, "How far is it to Baba's house?" I told him it was a long way away, about three hours, and, because it was such a long way, that they all had to be on their best behaviour. An hour later Cameron asked again "How far is it now?" "About two hours now." Another hour went by and he checked again. "About one hour," I said, and I smiled because they had all been so patient,

and so well behaved. The train was just coming into my mother's hometown when my nephew asked again, "How far is it to Baba's house now Uncle John?" "Only fifteen minutes now mate," I replied. My nephew's face dropped and his limbs went rigid in a paroxysmal fit; "Fifteen!?" he cried in genuine despair, "But last time you said it was one!?" I didn't know what to say. I felt terrible for not having understood his understanding of time, and for having repaid his good behaviour with such thoughtless treatment.

Cargo cults

In parts of Melanesia, in the late 19th and early 20th centuries, ritual practices that imitated the behaviour and artefacts of Western colonisers began to appear. While these practices may have served complex social functions designed to accommodate the shock of being confronted with Western culture and technology, they were also performed in the expectation that they would draw 'cargo', 'gifts from the gods', to the practitioners. Examples of these practices included maintaining jungle airstrips for aeroplanes that would never come, conducting military drills using sticks as guns, and building radio sets out of coconuts. The imitation of the surface features of Western behaviours and artefacts indicates the syncretism in this behaviour.

The dot-com bubble

Between 1997 and 2000, share markets boomed on the back of internet-related companies. Normally a company's stock price is a reflection of the value of its assets and anticipated earnings. In the 'dot-com' boom however, what often determined a company's value was whether or not its name had an 'e' before it and/or a 'dot-com' after it. The dot-com bubble is just one example of the speculative bubbles that have happened since a market for tulip bulbs opened in Holland in the 17th century. The fetishisation of the symbols 'e' and 'dot-com' indicates the syncretism in this behaviour.

The 'bigger fool' principle

A syncretic form of thought that contributes to the possibility of speculative bubbles is the egoistic belief that even though I may have foolishly paid too much for some 'thing', I am confident that there is a 'bigger fool than me out there somewhere' who is prepared to pay even more for it, so that I may expect to enjoy the profit that, to my mind, I so thoroughly deserve.

The Black-Scholes model

In 1973 Fisher Black and Myron Scholes published a paper on the pricing of stock options. Scholes and Robert Merton went on to win the 1997 Nobel Prize in economics for what became known as 'the Black-Scholes option pricing model'. Based on certain assumptions, and within certain limits, this model's set of partial differential equations made it possible to price an option to buy or sell a stock at some future date, risk-free. The model was so successful that, according to Ian Stewart, "By 2007, the international financial system was trading derivatives valued at one quadrillion dollars per year". [Derivatives are financial instruments that derive their value from other underlying values, such as stock prices, interest rates, or even other derivatives.] "This is 10 times the total worth, adjusted for inflation, of all products made by the world's manufacturing industries over the last century" (Stewart 2012). One of the assumptions underpinning this model is the relatively stable growth of underlying assets. However, between 2006 and 2008, US real estate values collapsed leading to a liquidity crisis, hedge-fund failures and taxpayer-funded bank rescues, otherwise known as the GFC, which subsequently contributed to European and US sovereign-debt crises, government austerity measures and a recession that persists in those parts of the world where people understood these equations well enough to plug values into them, but not well enough to recognise their limits. The 'bigger fool' principle, and blind faith in a set of equations, indicates the operation of syncretism here.

Sports cheats

The 2000 Summer Paralympics, held in Sydney, saw, perhaps, the most damaging and pointless example of cheating in the history of sport. In a games held under a motto that included “pride” as a core value, the gold medallists in the basketball for intellectually disabled men had their medals stripped from them when a team-member revealed that ten out of the team’s squad of twelve players had no disabilities whatsoever (BBC 2000). The will to win at any cost indicates the operation of syncretism. It also indicates, perhaps, that this team should have been awarded gold medals for the morally bankrupt.

Échappatoire

Adults’ day-to-day syncretic thinking is sustained by the self-unaware, or cynical, or just lazy use of figurative speech, slogans and catch phrases. The French refer to this (figuratively) as *échappatoire*, a use of language designed to provide a speaker with a ‘convenient escape from a difficult situation’. Examples of *échappatoire* can be found in the kinds of terms used in the popular debate on education policy, terms such as “Putting Students First” (an actual slogan), improved outcomes, raising standards, empowerment, autonomy, flexibility, best practice and teacher quality. These terms suit the operation of syncretic thought because, at first blush, they all appear to be self-evidently desirable, and, ultimately, because they are conveniently no clearer than the thing that they are supposed to illuminate (i.e. education policy). In teacher training, certain terms, like the zone of proximal development, scaffolding, feedback, formative assessment, and ‘what works’ can be used to realise *échappatoire*. *Échappatoire* can also be seen at work in the cultural and situational ‘types’ that we often use to account for our own or others’ behaviour. In this way abstract social and cultural ‘types’ that feature dimensions such as age, gender, ethnicity, religion, sect, tribe, nationality, socio-economic status and so on that aught to properly be the subjects of critique, become, instead, reified objects in the explanatory principles that underpin our world-views, and hence, of *échappatoire*.

7x13=28

There is an old comedy routine in which it is proven (in no less than three different ways!) that seven times thirteen equals twenty-eight. This routine, contrived to get a laugh at the comedian’s expense, is an example of syncretic thinking in that while it uses a mathematical understanding of arithmetic that is sophisticated, recognisable, plausible, systematic and repeatable, it is, thanks to its syncretic treatment of place value, nonetheless wrong. You can see this routine being done at http://www.youtube.com/watch?v=PyGFg0dHZBI.